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MECHANICS.

222. Proposed by W. J. GREENSTREET, Stroud, England.

Find the maximum angle of inclination to the line of greatest slope of a uniform rod resting on a rough inclined plane and capable of turning freely round a point on it.

II. Solution by the PROPOSER.

Let the point on the rod about which it turns be distant x and y from the ends $(x>y)$. Let α be the inclination of the plane, and θ the angle the rod in its initial position makes with the greatest line of slope. Let W be the weight per unit length of the rod. Consider the normal reactions as acting at the mid points, respectively, of AO, OB, where AB is the rod and O the pivot. μ is the coefficient of friction.

Resolving perpendicular to the plane for each part AO, OB,

$$R=\frac{x}{2}W\cos\alpha,\quad R'=\frac{y}{2}W\cos\alpha.$$

There is an unknown force at O. Take moments at O,

$$\mu.\frac{x}{2}R-\frac{x^2}{4}W\sin\alpha\sin\theta+\mu.\frac{y}{2}R'+\frac{y^2}{4}W\sin\alpha\sin\theta=0,$$

and substituting for R, R,

$$\mu\frac{x^2}{4}.W\cos\alpha+\mu\frac{y^2}{4}W\cos\alpha=\frac{W\sin\alpha\sin\theta}{4}(x^2-y^2),$$

$$\mu\cot\alpha\,(x^2+y^2)=\sin\theta\,(x^2-y^2).$$

$$\therefore\sin\theta=\mu\cot\alpha\frac{x^2+y^2}{x^2-y^2},\quad \theta=\sin^{-1}\left(\mu\cot\alpha.\frac{x^2+y^2}{x^2-y^2}\right).$$

224. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A steel clock spring $w=\frac{7}{8}$ inch wide, $t=\frac{1}{32}$ inch thick, is wound around an axle $d=\frac{1}{4}$ inch in diameter. Find the greatest available moment for running the clock, using a factor of safety $f=6$.

Solution by the PROPOSER.

Let $OH=r=$radius of curvature; $HL=y=$distance of any fiber from the gravity axis HH; $x=$length LM of this fiber; $MS=dx$; $RH=z=$distance between the neutral axis and the gravity axis; $\angle MRS=\phi$, $\angle AOB=\theta$; $s=$

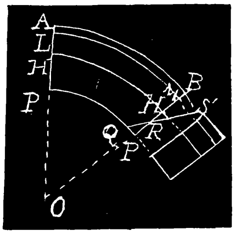
unit of tensile strength on cross-section of spring; E=Young's modulus; M=external bending moment acting on any area wy.

Then $x=LO.\theta=(r+y)\,\theta=(OH+HL)\,\theta$;

$dx=RM.\phi=(HR+HM)\phi=(z+y)\phi$.

Hooke's Law gives $dx/x=s/E$.

$\therefore s=\dfrac{Edx}{x}=\dfrac{E(z+y)\phi}{(r+y)\theta}=\dfrac{Ek(z+y)}{r+y}$, where k=the constant ϕ/θ.



For pure bending strain, the shear is zero. Then, since the normal stresses form a couple, their algebraic sum is zero.

$$\therefore \int swdy=0. \quad \therefore kEw\int\frac{(z+y)\,dy}{r+y}=0. \quad \therefore \int\frac{z}{r+y}dy+\int\frac{y}{r+y}dy=0.$$

$$\therefore z=-\frac{\displaystyle\int\frac{ydy}{r+y}}{\displaystyle\int\frac{dy}{r+y}}=-\frac{\displaystyle\int_{-\frac{1}{2}t}^{\frac{1}{2}t}\frac{ydx}{r+y}}{\displaystyle\int_{-\frac{1}{2}t}^{\frac{1}{2}t}\frac{dy}{r+y}}.$$

$$\therefore z=-\frac{t-r\log\left(\dfrac{2r+t}{2r-t}\right)}{\log\left(\dfrac{2r+t}{2r-t}\right)}=\tfrac{1}{2}d-\frac{t}{\log\left(\dfrac{d+t}{d-t}\right)}\ldots(1),\ \text{since } 2r=d.$$

Also $\displaystyle\int_{-\frac{1}{2}t}^{\frac{1}{2}t}(z+y)swdy=M$.

$$\therefore Ekw\int_{-\frac{1}{2}t}^{\frac{1}{2}t}\frac{(z+y)^2}{r+y}dy=M. \qquad \therefore k=\frac{M}{Ew\displaystyle\int_{-\frac{1}{2}t}^{\frac{1}{2}t}\frac{(z+y)^2dy}{r+y}}$$

$$s=\frac{Ek(y+z)}{r+y}=\frac{M(y+z)}{w(r+y)\displaystyle\int_{-\frac{1}{2}t}^{\frac{1}{2}t}\frac{(y+z)^2dy}{r+y}}.$$

$$\therefore s=\frac{M(t+2z)}{w(t+d)\left[\frac{1}{2}d-\dfrac{t}{\log\left(\dfrac{d+t}{d-t}\right)}\right]t}=\frac{M(t+2z)}{wtz(t+d)}\ldots(2).$$

This value of s is the maximum value. Now $d=\frac{1}{4}$, $t=\frac{1}{32}$, $w=\frac{7}{8}$.

From (1), $z=.00066$. $s=6416.9M$. The ultimate strength of hard steel is 240000 lb./in.^{2}.

$\therefore s=240000/f=240000/6=40000$ lb./in.^{2}

$\therefore 40000=6416.9M$. $\therefore M=6.2335$.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

159. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

Show that if the equation $y^3=2x^2-1$ be possible in integers, $y=24n^2-1$, or $2n^2-1$, and find three solutions.

Solution by the PROPOSER.

The equation may be written $(y+1)(y^2-y+1)=2x^2$.

Since y^2-y+1 is always odd, it is evident that $y+1$ must be even. Since $y^2-y+1=(y+1)(y-2)+3$ it is evident that $y+1$ and y^2-y+1 can have no common factor but 3. Therefore we have the following possibilities for y: $y=2\times3m^2-1$, or $y=2n^2-1$.

Since y^3 is represented by the form $2x^2-1$, 2 must be a quadratic residue of y. Therefore $y=8a\pm1$, and this is possible in the first expression only when $m=2n$. Then either $y=24n^2-1$ or $2n^2-1$.

Substituting these values of y in the original equation, we have

$$192n^4-24n^2+1=r^2 \text{ or } 4n^4-6n^2+3=r^2.$$

The first equation has the solution $n=0$ and 1 which give $y=-1$, $x=0$; $y=23$, $x=78$.

The second equation has the solution $n=1$, which gives $y=1$, $x=1$.

AVERAGE AND PROBABILITY.

198. Proposed by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

Find the average length of a hole at random through a given cylinder.

No solution of this problem has been received.

199. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

A circle is inscribed in a given square. Two points are taken at random within the square but without the circle. What is the chance the distance between the points does not exceed the side of the square?

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

This is the same as 196, but as there is a distance less than the side of the square when both points are taken one each in opposite corners, it is